# EXPRESSING MATHEMATICS AND MEASUREMENT IN ITHKUIL

I have determined that, using existing Ithkuil morpho-phonology, morphology and morpho-syntax, there really isn't an efficient way to state mathematical expressions in Ithkuil, including mathematical expressions involving units of measurement. Therefore, I've decided to create a formal sub-language within Ithkuil for dealing with mathematical expressions and units of measurement. You can think of this as a verbal analogy to the way that real-world written forms of mathematical expressions and rates of measurement require their own formal set of written symbols and symbolic notation rather than writing out mathematical expressions in words.

Note that I have chosen to maintain the existing informal centesimal system without a root for zero as described in Chapter 12 of the Ithkuil website, as a means for efficiently conveying an everyday "naïve" means of doing counting and very basic arithmetical operations consistent with the morpho-phonology, morphology, and morpho-syntax of the Ithkuil language.

At the same time, having a formal sub-language for higher mathematical expressions and measurement that follows its own internal morphological and syntactical rules allows for a succinct means of verbal mathematical expression and underscores the formalized, "special case" nature of mathematical expressions, again analagous to the formal written notation for mathematical expressions in real-world languages.

This treatise is in two parts; the first part focusing on mathematical expressions, the second part on units of mearurement.

## MATHEMATICAL EXPRESSIONS

Before introducing the new Ithkuil sub-language for formal mathematical expressions and measurement, I will first introduce the new Ithkuil roots, stems, and suffixes necessary for referencing higher mathematical concepts, terminology and expressions. These roots and stems can also, of course, be used to describe (to a limited extent) formal mathematical expressions in "standard" Ithkuil (i.e., without resorting to using the new mathematical sub-language).

**Ithkuil Roots, Stems, and SSD Derivatives Associated with Units of Measurement**

<table>
<tr><th colspan="4">-ŘŘ- ZERO</th></tr>
<tr><td colspan="2">1. zero as the empty-set / a set having no members; to mathematically have no quantity or measurable amount</td><td colspan="2">1. zero as placeholder for purposes of place-value notation/enumeration; to express zero as a placeholder for the purposes of place value notation/enumeration</td></tr>
<tr><td colspan="2">2. zero as the additive identity; to add (the) zero(-set) to an existing set or number</td><td colspan="2">2. zero as the cardinality of the empty set / the number of members of an empty set; to have no (i.e., zero) members in a set</td></tr>
<tr><td colspan="2">3. the zero-dimension, i.e., a Euclidean point; to have geometrically no length, area or volume, i.e., to be a Euclidean point</td><td colspan="2">3. a null value / a value for a parameter that is undefined and/or for which the expected/standard value(s) is/are inapplicable</td></tr>
<tr><td colspan="2">COMPLEMENTARY STEMS</td><td colspan="2">COMPLEMENTARY STEMS</td></tr>
<tr><td>Same as above with focus on stem as an abstract concept</td><td>Same as above with focus on stem in an applied context or equation</td><td>Same as above with focus on stem as an abstract concept</td><td>Same as above with focus on stem in an applied context or equation</td></tr>
</table>

<table>
<tr><th colspan="4">-RW- EXPRESSION OF MATHEMATICAL VALUE</th></tr>
<tr><td colspan="2">1. number; express numerically</td><td colspan="2">1. mathematical term; state as a mathematical term</td></tr>
<tr><td colspan="2">2. variable; express as a mathematical variable</td><td colspan="2">2. function; express as a function</td></tr>
<tr><td colspan="2">3. coefficient; express as a mathematical coefficient</td><td colspan="2">3. mathematical constant; express as/utilize a mathematical constant</td></tr>
<tr><td colspan="2">COMPLEMENTARY STEMS</td><td colspan="2">COMPLEMENTARY STEMS</td></tr>
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</table>

$SSD_1$ Derivatives for Informal Stem 1 of the above root:

| | |
|---|---|
| Degree 1 | integer |
| Degree 2 | negative integer |
| Degree 3 | positive integer |

| | |
|---|---|
| Degree 4 | Infinity ∞ |
| Degree 5 | counting/natural number |
| Degree 6 | irrational number |

| | |
|---|---|
| Degree 7 | rational number |
| Degree 8 | real number |
| Degree 9 | imaginary/complex number |

$SSD_1$ Derivatives for Informal Stem 2 refer to the specific hierarchy of variables in a formal mathematical expresson or equation, equivalent to Western mathematical notion x, y, z, . . . for variables in an algebraic equation:

| | |
|---|---|
| Degree 1 | $2^{nd}$-order variable (i.e., "y") |
| Degree 2 | $6^{th}$-order variable |
| Degree 3 | $4^{th}$-order variable |

| | |
|---|---|
| Degree 4 | $8^{th}$-order variable |
| Degree 5 | $1^{st}$-order variable (i.e., "x") |
| Degree 6 | $9^{th}$-order variable |

| | |
|---|---|
| Degree 7 | $5^{th}$-order variable |
| Degree 8 | $7^{th}$-order variable |
| Degree 9 | $3^{rd}$-order variable (i.e., "z") |

$SSD_1$ Derivatives for Informal Stem 3 refer to the specific hierarchy of coefficients in a formal mathematical expresson or equation, equivalent to Western mathematical notion a, b, c, . . . for coefficients in an algebraic expression/equation:

| | |
|---|---|
| Degree 1 | $2^{nd}$-order coefficient (i.e., "b") |
| Degree 2 | $6^{th}$-order coefficient |
| Degree 3 | $4^{th}$-order coefficient |

| | |
|---|---|
| Degree 4 | $8^{th}$-order coefficient |
| Degree 5 | $1^{st}$-order coefficient (i.e., "a") |
| Degree 6 | $9^{th}$-order coefficient |

| | |
|---|---|
| Degree 7 | $5^{th}$-order coefficient |
| Degree 8 | $7^{th}$-order coefficient |
| Degree 9 | $3^{rd}$-order coefficient (i.e., "c") |

$SSD_1$ Derivatives for Formal Stem 3:

| | |
|---|---|
| Degree 1 | *e* (i.e., the base of natural logarithms) |
| Degree 2 | λ (i.e., Conway's constant) |
| Degree 3 | φ (i.e., the "golden ratio") |

| | |
|---|---|
| Degree 4 | α (i.e., the first Feigenbaum constant) |
| Degree 5 | τ (i.e., tau = 2π) |
| Degree 6 | δ (i.e., the second Feigenbaum constant) |

| | |
|---|---|
| Degree 7 | γ (i.e., the Euler-Mascheroni constant) |
| Degree 8 | *K* (i.e., Khinchin's constant) |
| Degree 9 | *i/j* (i.e., the square-root of –1) |

$SSD_1$/5 derivative for Formal Stem 1: inverse of mathematical term or expression [the Ithkuil translation of "mathematical expression" of course derives from applying an appropriate Configuration to this stem].

$SSD_1$/5 derivative for Formal Stem 2: inverse of a mathematical function

Informal Stem 2 can also take the new **$DPD_1$/1** suffix or the **$DPD_1$/5** suffix (see below) to distinguish between the concepts of "dependent variable" versus "independent variable":

| -ňţ- | **DPD - Degree of Dependency** |
|---|---|
| Degree 1 | Objective value/identity/nature utterly dependent upon or determined by another entity |
| Degree 2 | Objective value/identity/nature mostly dependent upon or determined by another entity |
| Degree 3 | Objective value/identity/nature partially dependent upon or determined by another entity |
| Degree 4 | Objective value/identity/nature barely dependent upon or determined by another entity |
| Degree 5 | Utterly independent; value/identity/nature/efficacy completely self-determined; sui generis |
| Degree 6 | Efficacy/effectiveness/subjective value of X somewhat/barely determined/dependent upon another entity |
| Degree 7 | Efficacy/effectiveness/subjective value of X somewhat/partially determined/dependent upon another entity |
| Degree 8 | Efficacy/effectiveness/subjective value of X mostly determined/dependent upon another entity |
| Degree 9 | Efficacy/effectiveness/subjective value of X completely determined/dependent upon another entity |

| **-LY-** ARITHMETICAL/MATHEMATICAL OPERATIONS | | | |
|---|---|---|---|
| 1. add/subtract; act/process of adding/subtracting | | 1. sum/difference | |
| 2. multiply/divide; act/process of multiplying/dividing | | 2. product/quotient | |
| 3. express arithmetically/mathematically, apply arithmetical operation; arithmetical/mathematical expression | | 3. solve arithmetically/mathematically, calculate; arithmetical/mathematical calculation/computation | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| 1. add; act/process of adding | 1. subtract; act/process of subtracting | 1. sum | 1. difference |
| 2. multiply; act/process of multiplying | 2. divide; act/process of dividing | 2. product | 2. quotient |
| 3. apply arithmetical operation, express arithmetically; arithmetical expression | 3. apply mathematical operation, express mathematically; mathematical expression | 3. solve arithmetically, calculate arithmetically; arithmetical solution/calculation | 3. solve mathematically, calculate mathematically; mathematical solution/calculation |

Morphological derivatives of above stems: arithmetic, mathematics

$SSD_1$/5 derivative of Informal Stem 2: factorial, factorialization

| **-KŘ-** FRACTION / RATIO / RATE | | | |
|---|---|---|---|
| 1. fraction; to be/comprise a fraction of | | 1. ratio | |
| 2. factor; to be/determine a factor | | 2. trigonometric or hyperbolic ratio + inverse trigonometric or hyperbolic ratio | |
| 3. be proportional to, be in direct variance to + the constant/coefficient of proportionality; something proportional to something else + the constant/coefficient of proportionality | | 3. rate | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| 1. numerator | 1. denominator | 1. part-to-part ratio | 1. part-to-whole ratio |
| 2. greatest common factor | 2. least common multiple | 2. trigonometric or hyperbolic ratio | 2. inverse trigonometric or hyperbolic ratio |
| 3. be proportional to, be in direct variance to; something proportional to something else | 3. coefficient of proportionality / the constant of proportionality | 3. first term of a rate (i.e., the term subject to change in comparison to the fixed second term) | 3. second term of a rate (i.e., the fixed term against which the first term is subject to change; the term preceded in English by "per") |

$SSD_1$ Derivatives for Formal Stem 2, Pattern 2:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | cosine | Degree 4 | arcsine | Degree 7 | cosecant |
| Degree 2 | cotangent | Degree 5 | sine | Degree 8 | arctangent |
| Degree 3 | secant | Degree 6 | arccosine | Degree 9 | tangent |

$SSD_1$ Derivatives for Formal Stem 2, Pattern 3:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | hyperbolic cosine | Degree 4 | hyperbolic arcsine | Degree 7 | hyperbolic cosecant |
| Degree 2 | hyperbolic cotangent | Degree 5 | hyperbolic sine | Degree 8 | hyperbolic arctangent |
| Degree 3 | hyperbolic secant | Degree 6 | hyperbolic arccosine | Degree 9 | hyperbolic tangent |

<table>
<tr><td colspan="4">- KSṬ- MATHEMATICAL LIMIT / MATHEMATICAL DIFFERENTIATION & INTEGRATION</td></tr>
<tr><td colspan="2">1. numerical/mathematical limit (i.e., the limiting value of a function or summation of a series); to approach a limit in the output/dependent variable as the input/independent variable(s) of a function increases or decreases or as the process of summation of a series proceeds</td><td colspan="2">1. express/calculate an equation involving limiting values (i.e., a definite integral or a series)</td></tr>
<tr><td colspan="2">2. differential/derivative (i.e., the first differential coefficient of a function with respect to the independent variable); dy/dx or f '(x) or $\Delta y/\Delta x$ as $\Delta x \rightarrow 0$.</td><td colspan="2">2. express/calculate an equation involving differentiation/derivation</td></tr>
<tr><td colspan="2">3. integral/antiderivative</td><td colspan="2">3. express/calculate an equation involving an indefinite integral</td></tr>
<tr><td colspan="2">COMPLEMENTARY STEMS</td><td colspan="2">COMPLEMENTARY STEMS</td></tr>
<tr><td>1. the limiting value of a function); to approach a limit in the output/dependent variable as the input/independent variable(s) of a function increases or decreases</td><td>1. the limiting value of the summation of a series); to approach a limit as the process of summation of a series proceeds</td><td>1. express a mathematical integration or series; an expression/equation involving a definite integral or a series</td><td>1. calculate a mathematical integration or series; a calculation of a definite integral or a series</td></tr>
<tr><td>2. increment of the dependent variable, i.e., $\Delta y$</td><td>2. increment of the independent variable, i.e., $\Delta x$</td><td>2. express an equation involving differentiation/derivation; an expression/equation involving differentiation/derivation</td><td>2. calculate an equation involving differentiation/derivation; a calculation of a derivative / solving of a differential equation</td></tr>
<tr><td>3. indefinite integral / antiderivative</td><td>3. definite integral</td><td>3. express an equation involving an indefinite integral/antiderivative; an expression/equation involving an indefinite integral or antiderivative</td><td>3. calculate an indefinite integral or antiderivative; a calculation of an indefinite integral or antiderivative</td></tr>
</table>

Morphological derivatives: differential calculus, integral calculus

$SSD_1$ Derivatives for Formal and Informal Stem 2 refer to the hierarchy of derivatives:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | 3rd derivative; $d^3y/dx^3$ or f ' ' '(x) | Degree 4 | 9th derivative; $d^9y/dx^9$ or $f^{IX}(x)$ | Degree 7 | 6th derivative; $d^6y/dx^6$ or $f^{VI}(x)$ |
| Degree 2 | 7th derivative; $d^7y/dx^7$ or $f^{VII}(x)$ | Degree 5 | 2nd derivative; $d^2y/dx^2$ or f ' '(x) | Degree 8 | 8th derivative; $d^8y/dx^8$ or $f^{VIII}(x)$ |
| Degree 3 | 5th derivative; $d^5y/dx^5$ or $f^{V}(x)$ | Degree 6 | 10th derivative; $d^{10}y/dx^{10}$ or $f^{X}(x)$ | Degree 9 | 4th derivative; $d^4y/dx^4$ or $f^{IV}(x)$ |

| - **KSV**- EXPONENTIATION / LOGARITHM | | | |
|---|---|---|---|
| 1. act/process of exponentiation; raise a value to the power indicated by an exponent; multiply a value by itself for the number of iterations indicated by an exponent | | 1. express/calculate a value mathematically as a base and exponent, i.e., as a value raised to a particular power | |
| 2. act/process of finding the logarithm of a number given a particular base | | 2. express/calculate a value mathematically as a logarithm | |
| 3. act/process of finding the antilogarithm of a number, i.e., the number resulting from a base being raised to the power of a given exponent | | 3. express/calculate a value mathematically as an antilogarithm, (inverse function of a logarithm of a number) | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| 1. base value to be multiplied exponentially | 1. exponent or power of a base value | 1. express a mathematical value exponentially, i.e., as a value raised to a particular power | 1. calculate an exponential value |
| 2. logarithmic base (i.e., the fixed value which must be raised the number of times indicated by an exponent to result in a particular value) | 2. logarithm of a number (i.e., the exponent to which a base value must be raised to produce that number) | 2. express a mathematical value logarithmically | 2. calculate a mathematical value logarithmically |
| 3. antilogarithmic base | 3. the antilogarithm of a base and exponent, i.e., the number resulting from a base being raised to the power of a given exponent) | 3. express a mathematical value as an antilogarithm | 3. calculate an anti-logarithm |

| -**LBR**- SET, SEQUENCE, SERIES, MATRIX | | | |
|---|---|---|---|
| 1. element or member of a set; to be/comprise/make an element of member of a set | | 1. a mathematical matrix and its solution/value; to be/comprise a mathematical matrix | |
| 2. element or member of an arithmetic sequence; to be/comprise/make an element of member of an arithmetic sequence | | 2. an arithmetic series and its solution/value; to create/reference/obtain result from an arithmetic series (i.e., summation of an arithmetic sequence) | |
| 3. element or member of a geometric sequence; to be/comprise/make an element of member of a geometric sequence | | 3. a geometric series and its solution/value; to create/reference/obtain the result of a geometric series (i.e., summation of a geometric sequence) | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above 3 stems with focus on the element's/number's membership as an abstract concept | Same as above 3 stems with focus on the relationship of the element's/number's membership to a practical application. | 1. a mathematical matrix; to constitute/create a mathematical matrix | 1. solution/value of a mathematical matrix; to determine the result of a mathematical matrix |
| | | 2. an arithmetic series; to constitute/create an arithmetic series (i.e., summation of an arithmetic sequence) | 2. solution/value of an arithmetic series; to determine the result of an arithmetic series (i.e., summation of an arithmetic sequence) |
| | | 3. a geometric series; to constitute/create a geometric series (i.e., summation of a geometric sequence) | 3. solution/value of a geometric series; to determine the result a geometric series (i.e., summation of a geometric sequence) |

| **-MBR-** SCALAR / VECTOR / TENSOR | | | |
|---|---|---|---|
| 1. a scalar (i.e., first-degree tensor) | | 1. scalar product | |
| 2. a vector (i.e., a 2nd-degree tensor) | | 2. vector space | |
| 3. a (3rd- degree) tensor | | 3. tensor field | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| 1. scalar as magnitude of a vector | 1. scalar as direction of a vector | 1. dot product | 1. scalar quantity as inner product of two vectors |
| 2. Euclidean vector | 2. non-Euclidean vector | 2. Euclidean vector space | 2. non-Euclidean vector space |
| 3. tensor as n-dimensional array | 3. tensor as linear map | 3. tensor product of vector space | 3. point in a tensor field |

Existing roots/stems relevant to geometry:

angle, vertex: see -PṬ-
circle, ellipse: see -NR-
plane, cylinder: see -ṬM-
various/generic 2-dimensional outline shapes: -GM-, -JK-, -JPh-, -JTh-, -MŠ-
quadrilaterals: -ŇS-

For rectilinear n-sided two-dimensional forms (e.g., triangles, pentagons, octogons, etc.) see the –NNN suffix as used with the number-stems.

## The Mathematical Sub-Language

In order to verbalize and express decimals and other numbers beyond the natural numbers in a succinct manner, as well as express higher mathematical terms, equations, and rates, Ithkuil utilizes a "sub-language" whose expressions consist of an agglutinative string of consonant and vowel affixes, juxtaposed and linearly ordered in the same fashion as written numbers and/or spoken equations in English or other Western languages. Each such string is introduced by the word-initial syllable **ë-** being prefixed to the first word of the numerical or mathematical expression (since neither $\mathbf{V_L}$ in Slot II nor $\mathbf{V_r}$ in Slot IV can have this value).

Each word in the expression is stressed on the penultimate syllable except the last word of the expression which has ultimate syllabic stress and takes high or rising tone in order to indicate termination of the expression. If the expression in the sub-language is to be placed within a normal Ithkuil sentence, then the expression is treated like a noun or case-frame and is prefaced by the carrier root -**P**- in Formal Stem 2 to show the case and any other morphology associated with the expression. When prefaced by the carrier root, the **ë-** prefix on the first word of the expression is unnecessary. However, if the mathematical expression functions as the equivalent of a noun (or case-frame) in the OBLIQUE case, then one may retain the **ë-** prefix and delete the carrier-root. Either way, the mathematical expression will retain tonic stress and high or rising tone on its final syllable to indicate the end of the expression.

The numerical base employed for the sub-language is base-12 due to its relatively high number of whole-number factors. While the author would have preferred to use base-60 due to its even higher number of factors, creating a separate consonantal form for 60 different numbers would have depleted the available consonant inventory for use in the sub-language.

The number affixes are as follows:

| **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **A** | **B** | **.** | **τ** | ***e*** | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ň | l | k | ţ | p | s | q | n | f | x | m | t | š | xh | ç | ļ |

The symbols "A" and "B" represent ten and eleven in a base-12 number system for the purposes of this presentation. The period represents the "duodecimal" point, equivalent to a decimal point in base-10. The symbol τ (tau) represents the ratio of the radius of a circle to its circumference, equivalent to 2π. The symbol e represents Euler's constant, the base of natural logarithms. The ∞ symbol represents infinity.

The thirteen consonant forms for the numbers zero through B, plus the duodecimal point, are strung together linearly just as Arabic numerals are from left to right, utilizing the neutral vowel **ë** where necessary to accommodate phonotactic/euphonic constraints.

Rather than repeating the consonant **-ň-** multiple times, a string of zeroes within a larger number is shown by the vowel **o** immediately preceding a particular number indicates a string of zeroes of that particular number in length.

A negative number is indicated by prefixing **-r-** immediately to the first consonant of the number.

Examples:

¯**ëll** = 11 = 13 (base 10)

¯**ënëëň** = 70 = 84 (base 10)

¯**ëtf** = B8 = 140 (base 10)

¯**ërkëkšëëp** (or ¯**ërëkkëëšt**) = –22.4 = –26.333… (base 10)

¯**ëmlësnëëpf** (or ¯**ëmëlsnëëpf**) = A15748 = 2518760 (base 10)

¯**ëssëssësslëëss** = 555555.55 = 1357265.38 (base 10)

¯**ërfqëtmënopţëësk** = –76BA70000352 = –5633871004142 (base 10)

Likewise, common mathematical operators and various mathematical expessions of a number or variable are indicated by consonantal or vocalic affixes. These elements each make reference to the number or variable immediately following, as shown in the examples following the charts.

| | vocalic form | consonant form |
|---|---|---|
| + ; plus X; add X | i | tw |
| · ; multiplied by X | a | kw |
| 1/X; reciprocal of X; $X^{-1}$ | ô | ř |
| mod X | üa | zy |
| = ; is equal to X | â | mw |
| ≠ ; not equal to X | aù | cw |
| ≈ ; approx. equal to X | aì | čw |
| ± ; plus or minus X | eù | cy |
| \|X\|; the absolute value of X | eì | zw |
| > ; is greater than X | oe | sy |
| < ; is less than X | eo | šy |
| ≥ ; is no less than X | ae | vw |
| ≤ ; is no greater than X | ao | dhw |
| ≅ ; is congruent to X | eö | vy |
| ≡ ; is defined as X | öa | dhy |
| $a_X$ ; sub X; X as subscript | oi | ry |

| | vocalic form | consonant form |
|---|---|---|
| raised to the $X^{th}$ power; $a^X$ | e | hl |
| log X (to base *e* unless base specified) | ö | hr |
| to the base X [logarithmic base] | ü | pw |
| √ ; square root of X | ê | qw |
| $X^{th}$ root | u | py |
| ...root of X | ou | ky |
| ∝ ; is proportional to x | ea | ty |
| *i*·X; X times √-1 | oa | rw |
| sin X | ai | hm |
| cos X | ei | hn |
| tan X | ui | hw |
| sinh X | au | sw |
| cosh X | eu | šw |
| tanh X | iu | çw |
| inverse of X | öu | řw |
| X!; X factorial | öi | řy |

| | |
|---|---|
| begin parenthesis 1st level | t’ |
| begin parenthesis 2nd level | p’ |

| | |
|---|---|
| end parenthesis 1st level | ut’ |
| end parenthesis 2nd level | up’ |

| | |
|---|---|
| begin parenthesis 3rd level | k’ |
| begin parenthesis 4th level | q’ |

| | |
|---|---|
| end parenthesis 3rd level | uk’ |
| end parenthesis 4th level | uq’ |

| | |
|---|---|
| 1st coefficient; e.g., *a* | c |
| 2nd coefficient; e.g., *b* | č |
| 3rd coefficient; e.g., *c* | ż |
| 4th coefficient | c’ |
| unknown constants, e.g., *C, n, m* | j, $p^h$, $q^h$ |
| dependent variable; *y* | z |

| | |
|---|---|
| 1st independent variable; *x* | v |
| 2nd independent variable | dh |
| 3rd independent variable | ž |
| 4th independent variable | č’ |
| Temporal variable; *t* | $c^h$ |
| the radius of a circle; *r* | $č^h$ |

Additional coefficients can be created using the sub-X affix, e.g., **-coil, -čoik**, equivalent to saying $a_1$, $b_2$.

Consonants denoting coefficients and variables can also be geminated to give forms equivalent to saying “a prime” or “x-prime”, e.g., **-cc-, -vv-** “ *a′, x′* ”

Subtraction is shown by addition of a negative number; division is shown by multiplication of the reciprocal of a number. As in Western mathematical notation, multiplication of entities other than two numbers (e.g., variables, coefficients, a number and a variable, etc.) may be shown by simply juxtaposing the entities without the **-a-** affix, as long as the results are unambiguous.

Any juxtaposed vocalic affixes are separated by an epenthetic intervocalic infix **-h-**. Examples:

**ëçet’oaxhôkut’ irël ¯âň**

$e^{i\pi} - 1 = 0$

**ëz âhnët’ëcvekisdhut’ ¯àiřëv**

$$y = \frac{\cos(ax^2 + 5b)}{\sin x}$$

**ëhrujuovüc ¯ârjövüüc** $\log_a \sqrt[n]{x} = \frac{1}{n}\log_a x$

**Ieksţawél ëhêvt’ëkvirlut’ ¯t’ëvekivíl.** OR **Ieksţawél epál êvt’ëkvirlut’ ¯t’ëvekivíl.**

*Find the derivative of the equation* $\sqrt{x}(2x-1)(x^2+x+1)$ .
(Note that in this example, it is unnecessary to indicate the final closed pararenthesis using -**ut’**, since its parenthetical group is the last term in the expression and it contains no lower-order nested parenthetical groups.)

Additional affixes are shown below:
* (vocalic affixes with asterisks indicate where the X (and Y) elements are placed relative to the affix)

| | | |
|---|---|---|
| limit as X approaches Y | *awa* | — |
| for the interval beginning w/ X | *ay(ë) | my |
| for the interval beginning w/ X and ending w/ Y | *aya* | — |
| function of X; f(X) | owa* | xw |
| increment of change in X; $\Delta x$ | oya* | xhw |
| $\sum$ X; summation of X | awo* | lw |
| $\prod$ X; product of X | ayo* | ly |
| from the starting value X | *oy(ë) | ny |

| | | |
|---|---|---|
| dy; The Xth differential coefficient of Y; the Xth derivative of Y | *ia* | — |
| dx; (differential or integral ) with respect to X | ua* | ňw |
| $\partial$y: The Xth partial differential coefficient of; the Xth partial derivative of | *io* | — |
| $\partial$x; (partial differential of . . . ) with respect to X | uo* | ňy |
| $\int$ ; integral of X; (indefinite or definite) | *ie* | ţw |
| $\iint$ ; double integral | *iö* | ļw |
| $\iiint$ ; triple integral | *iù* | ļy |
| $\oint$ ; contour integral of X | *ue* | ţy |

| | | |
|---|---|---|
| from the starting value X to the ending value Y | *oye* | — |
| for X number of iterations | owe* | nw |

| | | |
|---|---|---|
| ∯ ; closed surface integral | *uö* | — |
| ∰ ; closed volume integral | *uì* | — |

More examples:

**ëţwëcevuav ¯âcevöçüüc**

$$\int a^x \mathrm{d}x = a^x \cdot \log_a \mathrm{e}$$

**ëç ¯âjawaḷt'ëlirjut'éj**

$$\mathrm{e} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$$

**ëlwëjekvejkoyep ¯âpvekixveţilpvép**

$$\sum_{n=2}^{4} n^2 x^n = 4x^2 + 9x^3 + 16x^4$$

**Pss uicál ëliazuav ¯âjvet'ëjirlút'. Üapšal ëkiazuav âjt'ëjirlut' ¯avet'ëjirkút'.**

*If* $\frac{\mathrm{d}y}{\mathrm{d}x} = nx^{n-1}$ *then* $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = n(n-1)x^{n-2}$.

LITERALLY: 'Posit that there exists [1st equation](-OBL). It occurs as a consequence [2nd equation](-OBL).'

Affixes associated with set theory, logic, and intervals are shown below. The asterisks indicate where consonantal values referring to variables, labels, numbers, coefficients, etc. are to be placed:

| | |
|---|---|
| { } ; the set A | awe* |
| \| ; such that | aye* |
| ∪ ; union of A and B | *ewa* |
| ∩ ; intersection of A and B | *eya* |
| ⊆ ; A is a subset of B | *ewo* |
| ⊂ ; A is a proper/strict subset of B | *eyo* |
| ⊄ ; A is not a subset of B | *e'o* |
| ⊇ ; A is a superset of B | *ewe* |
| ⊃ ; A is a proper/strict superset of B | *eye* |
| ⊅ ; A is not a superset of B | *e'e* |
| ℙ ; power set of A; all subsets of A | awi* |
| $A^c$ ; complement of set A | ayu* |
| \ ; relative complement of A | aro* |
| ⊖ ; symmetric difference of A and B | *a'o* |
| ∈ ; element of A | ara* |
| ∉ ; not an element of A | a'a* |

| | |
|---|---|
| ∧ ; and | aru* |
| ∨ ; or | a'u* |
| ¬ ; not, negation | a'i* |
| ⊕ ; either A or B but not both; xor | *a'ru* |
| ⇒ ; implies | owi* |
| ⇔ ; implies...& vice-versa ; iff | oyu* |
| ∀ ; for all | iwa* |
| ∃ ; there exists | ora* |
| ∄ ; there does not exist | o'a* |
| ∴ ; therefore | i'a* |
| ∵ ; because/since | u'a* |
| [ ] ; closed interval between a and b | *e'a* |
| ] [ ; open interval between a and b | *era* |
| [ [ ; right-open interval b/w a and b | *ero* |
| ] ] ; left-open interval b/w a and b | *ere* |
| ≺ ; a is a predecessor of b | *e'i* |

| | |
|---|---|
| # ; cardinality of set A | ari* |
| ⊍ ; disjoint union of A and B | *are* |
| ⊔ ; disjoint intersection of A and B | *a'e* |
| $\mathbb{N}_0$ ; natural/whole numbers with zero | $k^h$ |
| $\mathbb{N}_1$ ; natural/whole numbers w/o zero | $t^h$ |
| ℤ ; integer numbers set | $p^hw$ |
| ℚ ; rational numbers set | $k^hw$ |

| | |
|---|---|
| ≻ ; a is a successor of b | *e'u* |
| ⌊ ⌋ ; biggest whole number ≤ X | iwe* |
| ⌈ ⌉ ; smallest whole number ≥ X | uye* |
| ℝ ; real numbers set | $t^hw$ |
| ℂ ; complex numbers set | $q^hw$ |
| Ø ; the empty set | $lp^h$ |
| 𝕌 ; the universal set | $lq^h$ |

| PHYSICAL CONSTANT | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN |
|---|---|---|
| Avogadro's number | bm | $SSD_1$/1 of Formal P1/S3 of -**C**- |
| reduced Planck constant | gw | $SSD_1$/2 of Formal P1/S3 of -**C-** |
| Coulomb constant | by | $SSD_1$/3 of Formal P1/S3 of -**C**- |
| Universal gas constant | $nt^h$ | $SSD_1$/4 of Formal P1/S3 of -**C**- |
| speed of light in a vacuum | bw | $SSD_1$/5 of Formal P1/S3 of -**C**- |
| Rydberg constant | $mp^h$ | $SSD_1$/6 of Formal P1/S3 of -**C**- |
| Stefan-Boltzmann constant | dy | $SSD_1$/7 of Formal P1/S3 of -**C**- |
| universal gravitational constant | dw | $SSD_1$/8 of Formal P1/S3 of -**C**- |
| Acceleration due to gravity | gy | $SSD_1$/9 of Formal P1/S3 of -**C**- |

## UNITS OF MEASUREMENT

Units of measurement in Ithkuil are conceptual only and do not, in and of themselves, specify or correspond to any Western or real-world arbitrary unit such as meters, degrees, seconds, etc. As examples, the Ithkuil stems meaning "basic unit of incremental time" and "basic unit of incremental temperature" mean only those things; they do not mean "second" or "kelvin".

Nevertheless, in order to express actual meaningful measurements, a means for referencing real-world arbitrary units of measurement is necessary. To do this, Ithkuil unit-of-measurement stems utilize the SSD derivational suffix to indicate correspondence to such arbitrary units.

Ithkuil's mathematical sub-language also allows for the expression of measurements, the details of which will be given later, after the section below indicating the roots, stems, and SSD suffix designations associated with units of measurement.

**Ithkuil Roots, Stems, and SSD Derivatives Associated with Units of Measurement**

| -**KP$^{h}$**- MEASUREMENT OF SPATIO-TEMPORAL OR DIMENSIONAL PROPERTY | | | |
|---|---|---|---|
| 1. measurement of (quasi-)linear dimensionality, e.g., length, height, width, etc.; to measure the length of | | 1. measure/measurement of temporal duration; to time something | |
| 2. measurement of a planar angle; to measure a planar angle | | 2. measure/measurement of cyclic/periodic frequency = number of cycles per unit of time | |
| 3. measurement of a volumetric (solid) angle | | 3. measure/measurement of recurring but non-cyclic/aperiodic activity or events = number of events per unit of time | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$ Derivatives for Informal Stem 1 for Patterns 1, 2, and 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | Planck length / in Planck lengths | Degree 4 | millimeter / in mm | Degree 7 | astronomical unit / in AUs |
| Degree 2 | in angstrom / in angstroms | Degree 5 | meter / in meters | Degree 8 | light-year / in light years |
| Degree 3 | micron / in microns | Degree 6 | kilometer / in km | Degree 9 | parsec / in parsecs |

$SSD_2$ Derivatives for Informal Stem 1 for Patterns 1, 2, and 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | inch / in inches | Degree 4 | fathom / in fathoms | Degree 7 | mile / in miles |
| Degree 2 | foot / in feet | Degree 5 | rod / in rods | Degree 8 | nautical mile / in nautical miles |
| Degree 3 | yard / in yards | Degree 6 | furlong / in furlongs | Degree 9 | league / in leagues |

$SSD_1$ Derivatives for Informal Stem 2 for Patterns 1, 2, and 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | point / in points | Degree 4 | mil / in mils | Degree 7 | second (of arc) / in seconds |
| Degree 2 | hour angle / in hour angles | Degree 5 | radian / in radians | Degree 8 | minute (of arc) / in minutes |
| Degree 3 | grad / in grads | Degree 6 | sextant / in sextants | Degree 9 | degree / in degrees |

$SSD_1$/5 derivative for Informal Stem 3 in Pattern 3 = steradian

$SSD_1$ Derivatives for Formal Stem 1 for Patterns 1, 2, and 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | Planck time unit / in Planck t.u. | degree 4 | minute / in minutes | degree 7 | week / in weeks |
| Degree 2 | millisecond / in milliseconds | degree 5 | hour / in hours | degree 8 | month / in months |
| Degree 3 | second / in seconds | degree 6 | day (24-hrs) / in days | degree 9 | calendar year / in c. years |

$SSD_2$ Derivatives for Formal Stem 1 for Patterns 1, 2, and 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | sidereal year / in s. yrs | degree 4 | millenium / in millenia | degree 7 | era (= 10 epochs) / in eras |
| Degree 2 | decade / in decades | degree 5 | age (= 1 million yrs.) / in ages | degree 8 | galactic year / in g.yrs. |
| Degree 3 | century / in centuries | degree 6 | epoch (= 10 ages) / in epochs | degree 9 | eon (= 5 eras) / in eons |

$SSD_1$/5 derivative for Formal Stem 3 in Pattern 1 = hertz

$SSD_1$ Derivatives for Formal Stem 3 for Patterns 1, 2, and 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | rutherford / in rutherfords | Degree 5 | becquerel / in becquerels | Degree 9 | curie / in curies |

| -**RK$^h$**- AREA AND VOLUMETRIC MEASUREMENT / MEASUREMENT OF VELOCITY & ACCELERATION | | | |
|---|---|---|---|
| 1. measurement of (quasi-)planar area or surface; measure a (quasi-)planar area or surface | | 1. measurement of the rate of speed/velocity; measure the speed/velocity of an object (= distance divided by time) | |
| 2. measurement of 3-dimensional volume; measure a 3-dimensional volume | | 2. measurement of the rate of acceleration; measure the acceleration of an object (= distance per unit of time squared) | |
| 3. measurement of hyperdimensional volume; measure a hyperdimensional volume | | 3. measure/measurement of the rate of other phenomena utilizing units of distance per units of time | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$ Derivatives for Informal Stem 1, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | barn / in barns | Degree 4 | sq. meter / in sq. meters | Degree 7 | tetrad / in tetrads |
| Degree 2 | Sq. millimeter / in sq. mm | Degree 5 | hectare / in hectares | Degree 8 | hectad / in hectads |
| Degree 3 | Sq. centimeter / in sq. cm | Degree 6 | sq. kilometer / in sq. km | Degree 9 | myriad / in myriads |

$SSD_2$ Derivatives for Informal Stem 1, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | sq. inch / in sq. inches | Degree 4 | square / in squares | Degree 7 | acre / in acres |
| Degree 2 | sq. foot / in sq. feet | Degree 5 | sq. mile / in sq. miles | Degree 8 | virgate / in virgates |
| Degree 3 | sq. yard / in sq. yards | Degree 6 | sq. perch or sq. rod / in sq/ perches or rods | Degree 9 | township / in townships |

$SSD_1$ Derivatives for Informal Stem 2, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | minim / in minims | Degree 4 | cubic micron / in $\mu^3$ | Degree 7 | fluid ounce / in fl. oz. |
| Degree 2 | fluid dram / in fl. dr. | Degree 5 | milliliter / in milliliters | Degree 8 | gallon / in gallons |
| Degree 3 | teaspoon / in tsp. | Degree 6 | liter / in liters | Degree 9 | barrel |

$SSD_1$ Derivatives for Formal Stem 1, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | bubnoff unit | Degree 4 | radians per second | Degree 7 | kilometers per hour |
| Degree 2 | inch per second | Degree 5 | meters per second | Degree 8 | knot |
| Degree 3 | foot per second | Degree 6 | kilometers per second | Degree 9 | miles per hour |

$SSD_1$ Derivatives for Formal Stem 2, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | bubnoff unit$^2$ | Degree 4 | radians per second$^2$ | Degree 7 | kilometers per hour$^2$ |
| Degree 2 | inch per second$^2$ | Degree 5 | meters per second$^2$ | Degree 8 | knot per hour |
| Degree 3 | foot per second$^2$ | Degree 6 | kilometers per second$^2$ | Degree 9 | miles per hour$^2$ |

$SSD_1$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | angular velocity or angular frequency, as measured in units of planar angle measurement per unit of time |
| Degree 2 | Kinematic viscosity or diffusivity coefficient, as measured in distance squared per unit of time |
| Degree 3 | Snap or jounce, as measured in distance per unit of time to the fourth power |
| Degree 5 | Jerk, jolt, surge or lurch, as measured in distance per unit of time cubed |
| Degree 7 | Volumetric flow, as measured in distance cubed per unit of time |
| Degree 9 | Spread rate by volume, as measured in distance cubed per distance squared |

| **-KPL-** MEASUREMENT OF ENERGY/FORCE/PRESSURE/POWER | | | |
|---|---|---|---|
| 1. measure/measurement of mass | | 1. measure/measurement of pressure = (mass x (distance/time$^2$))/distance$^2$ | |
| 2. measure/measurement of energy/work = (mass x distance$^2$)/time$^2$ | | 2. measure/measurement of power = mass x distance$^2$/time$^3$ | |
| 3. measure/measurement of force = mass x distance/time$^2$ | | 3. measure/measurement of the rate of other phenomena utilizing units of energy/force/pressure/power | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$ Derivatives for Informal Stem 1, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | Planck mass / in Planck masses | Degree 4 | gram / in grams | Degree 7 | pound / in lbs. |
| Degree 2 | dalton / in daltons | Degree 5 | kilogram / in kg | Degree 8 | (short) ton (= 2000 lbs.) |
| Degree 3 | grain / in grains | Degree 6 | metric tonne / in m. tonnes | Degree 9 | solar mass / in solar masses |

$SSD_1$ Derivatives for Informal Stem 2, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | Planck energy / in $E_p$ | Degree 4 | hartree / in hartrees | Degree 7 | calorie / in calories |
| Degree 2 | electronvolt / in eV | Degree 5 | joule / in joules | Degree 8 | thermie / in thermies |
| Degree 3 | erg / in ergs | Degree 6 | therm / in therms | Degree 9 | quad / in quads |

$SSD_1$ Derivatives for Informal Stem 3, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | Planck force / in $F_p$ | Degree 4 | pound-force / in lbf | Degree 7 | sthène / in sthènes |
| Degree 2 | dyne / in dynes | Degree 5 | newton / in newtons | Degree 8 | kip / in kips |
| Degree 3 | poundal / in poundals | Degree 6 | kilipond / in kiliponds | Degree 9 | ton-force / in ton-forces |

$SSD_1$ Derivatives for Formal Stem 1, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | bar / in bars | Degree 4 | torr / in torrs | Degree 7 | millimeters of mercury / in millimeters of mercury |
| Degree 2 | barye / in baryes | Degree 5 | pascal / in pascals | Degree 8 | inches of mercury / in inches of mercury |
| Degree 3 | pounds per sq. inch / in lbs. per sq. inch | Degree 6 | pièze / in pièzes | Degree 9 | standard atmosphere / in standard atmospheres |

$SSD_1$ Derivatives for Formal Stem 2, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | ton of refrigeration | Degree 5 | watt / in watts | Degree 9 | horsepower |

$SSD_1$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | line density, as measured by mass per distance |
| Degree 2 | volumetric density, as measured by mass per volume |
| Degree 3 | specific volume, as measured by volume per mass |
| Degree 4 | spread rate, as measured by mass per area |
| Degree 5 | area density, as measured by mass per area |
| Degree 6 | momentum, as measured by mass times distance/time |
| Degree 7 | angular momentum, as measured by mass times distance$^2$/time |
| Degree 8 | thrust, as measured by mass times distance/time$^2$ |
| Degree 9 | torque or moment, as measured by mass times distance$^2$/time$^2$ |

$SSD_2$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | yank, as measured by mass times distance/time$^3$ |
| Degree 2 | rate of absorbed dose of ionizing radiation, as measured by energy per mass/time |
| Degree 3 | specific energy, as measured by energy per unit mass (e.g., joules per kg, sieverts, grays, rads) |
| Degree 4 | radiant exposure of a surface, energy distribution over a surface, insolation, solar radiation; as measured by energy per square distance |
| Degree 5 | energy density, as measured by energy per cubic distance |
| Degree 6 | surface tension, as measured by force per distance |
| Degree 7 | stiffness, as measured by force per distance |
| Degree 8 | dynamic viscosity, as measured by pressure multiplied by time (e.g., poises, poiseuilles) |
| Degree 9 | acoustic impedance, as measured by pressure multiplied by time per distance (e.g., rayls) |

Additional SSD Derivatives for Formal Stem 3, Pattern 1, formed by suffixing the following $SSD_1$ affixes to the $SSD_1$/5 affix:

| | |
|---|---|
| Degree 1 | spectral flux by frequency, as measured by power per cycle/time |
| Degree 2 | spectral flux by wavelength, as measured by power/distance |
| Degree 3 | Heat flux density, irradiance, radiant exitance, radiosity; as measured by power/distance$^2$ |
| Degree 4 | Spectral exitance/radiosity/irradiance by frequency; strength of radio wave emission, as measured by power/distance$^2$ per cycle/time (e.g., in janskys) |
| Degree 5 | Spectral exitance/radiosity/irradiance by wavelength, as measured by power/distance$^2$ per distance |
| Degree 6 | radiant intensity, as measured by power/distance$^2$ per steradian |
| Degree 7 | spectral intensity, as measured by power/distance$^2$ per steradian per cycle/time |
| Degree 8 | radiance, as measured by power/distance$^2$ per steradian per distance$^2$ |
| Degree 9 | Spectral radiance by frequency, as measured by power/distance$^2$ per steradian per distance$^2$ per cycle/time |

| **-JG-** MEASUREMENT OF TEMPERATURE & THERMODYNAMIC PHENOMENA | | | |
|---|---|---|---|
| 1. measure/measurement of temperature | | 1. measure/measurement of specific heat capacity or specific entropy, as measured in energy per mass times temperature | |
| 2. measure/measurement of heat capacity or entropy, as measured in energy per unit temperature | | 2. measure/measurement of thermal resistance, as measured in temperature per unit of power | |
| 3. measure/measurement of enthalpy, as measured in units of energy | | 3. measure/measurement of thermal conductivity, as measured in power per distance times temperature | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$ Derivatives for Informal Stem 1, Pattern 3 of the above root:

| | |
|---|---|
| Degree 1 | Planck temperature / in $T_p$ |
| Degree 2 | degrees Newton / in °N |
| Degree 3 | degrees Celsius / in °C |

| | |
|---|---|
| Degree 4 | degrees Rankine / in °R |
| Degree 5 | kelvin / in kelvins |
| Degree 6 | degrees Delisle / in °D |

| | |
|---|---|
| Degree 7 | degrees Fahrenheit / in °F |
| Degree 8 | degrees Rømer / in °Rø |
| Degree 9 | degrees Réaumur / in °Ré |

| -**MP$^{h}$**- MEASUREMENT OF ELECTRO-MAGNETIC PHENOMENA | | | |
|---|---|---|---|
| 1. measure/measurement of electric current | | 1. measure/measurement of magnetic flux, as measured by energy per unit current = mass times distance$^2$ divided by time$^2$ times current | |
| 2. measure/measurement of electrical charge, as measured by electric current times unit of time | | 2. measure/measurement of electrical capacitance, as measured by time$^4$ times current$^2$ per mass times distance$^2$ | |
| 3. measure/measurement of electrical potential difference and electromotive force, as measured in mass times distance$^2$ divided by unit of current multiplied by time$^3$ | | 3. measure/measurement of electrical inductance, as measured by mass times distance$^2$ divided by time$^2$ times current$^2$ | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$ Derivatives for Informal Stem 2, Pattern 3 of the above root:

| Degree 1 | biot (or abampere) | Degree 5 | ampere | Degree 9 | statampere |
|---|---|---|---|---|---|

$SSD_1$ Derivatives for Informal Stem 2, Pattern 3 of the above root:

| Degree 1 | franklin | Degree 5 | coulomb | Degree 9 | statcoulomb |
|---|---|---|---|---|---|

$SSD_1$ Derivatives for Informal Stem 3, Pattern 3 of the above root:

| Degree 1 | abvolt | Degree 5 | volt | Degree 9 | statvolt |
|---|---|---|---|---|---|

$SSD_1$ Derivatives for Formal Stem 1, Pattern 3 of the above root:

| Degree 1 | unit pole | Degree 5 | weber | Degree 9 | maxwell (or line) |
|---|---|---|---|---|---|

$SSD_1$ Derivatives for Formal Stem 2, Pattern 3 of the above root:

| Degree 1 | abfarad | Degree 5 | farad | Degree 9 | statfarad |
|---|---|---|---|---|---|

$SSD_1$ Derivatives for Formal Stem 3, Pattern 3 of the above root:

| Degree 1 | abhenry | Degree 5 | henry | Degree 9 | stathenry |
|---|---|---|---|---|---|

| -**N̆Č$^{h}$**- MEASUREMENT OF SECONDARY ELECTRO-MAGNETIC PHENOMENA | | | |
|---|---|---|---|
| 1. measure/measurement of electrical resistance of circuits, as measured by mass times distance$^2$ divided by the quantity of time$^3$ times current$^2$ | | 1. measure/measurement of magnetic flux density or B-magnetic field strength, as measured by force per electric current per distance | |
| 2. measure/measurement of electrical conductance, as measured by time$^3$ times current$^2$ divided by the quantity of mass times distance$^2$ | | 2. measure/measurement of relative difference, i.e., ratio between two values of a physical quantity, e.g. power, intensity, current, voltage, loss or gain of an electronic signal, etc. | |
| 3. measure/measurement of magnetomotive force, as measured by a unit of current flowing in a single-turn loop in a vacuum | | 3. measure/measurement of the rate of other electro-magnetic phenomena utilizing units of electro-magnetic measurement along with units of distance, time, or mass | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$ Derivatives for Informal Stem 1, Pattern 3 of the above root:

| Degree 1 | abohm | Degree 5 | ohm | Degree 9 | statohm |
|---|---|---|---|---|---|

$SSD_1$ Derivatives for Informal Stem 2, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | abmho | Degree 5 | siemens (or mho) | Degree 9 | statmho |

$SSD_1$ Derivatives for Informal Stem 3, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | abampere turn | Degree 5 | ampere-turn | Degree 9 | gilbert |

$SSD_1$ Derivatives for Formal Stem 1, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | gauss (or abtesla) | Degree 5 | tesla | Degree 9 | stattesla |

$SSD_1/5$ derivative for Formal Stem 2, Pattern 3 of the above root: neper

$SSD_1$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | reciprocal inductance, reluctance, as measured by the reciprocal of mass times distance$^2$ divided by time$^2$ times current$^2$ (e.g., siemens or ohm$^{-1}$) |
| Degree 2 | electric field, as measured by force per unit charge or electrical potential difference per distance (e.g., volts per mtr or newtons per coulomb) |
| Degree 3 | electric displacement field, polarization vector, as measured by charge per distance squared (e.g., coulomb per sq. meter) |
| Degree 4 | electric charge density, as measured by charge per distance cubed (e.g., coulomb per cu. meter) |
| Degree 5 | electric current density, as measured by current per distance squared (e.g., amperes per sq. meter) |
| Degree 6 | electrical resistivity, as measured by electrical resistance times distance (e.g., ohm meter) |
| Degree 7 | electrical conductivity, as measured by electrical conductance per distance (e.g., siemens per meter) |
| Degree 8 | electromagnetic emittivity, as measured by electrical capacitance per distance (e.g., farads per meter) |
| Degree 9 | electromagnetic permeability, as measured by electrical inductance per distance (e.g., henrys per meter) |

$SSD_2$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | electric elastance, as measured by the reciprocal of electrical capacitance (e.g., daraf = reciprocal of farad) |
| Degree 5 | H-magnetic field strength, as measured by current per distance (e.g., oersteds or amperes per meter) |
| Degree 9 | exposure to ionizing radiation, as measured by charge per mass (e.g., coulombs per kilogram) |

| -**CTW**- MEASUREMENT OF ILLUMINATION | | | |
|---|---|---|---|
| 1. measure/measurement of luminous intensity | | 1. measure/measurement of illuminance, luminous exitance or emittance, as measured by luminous intensity per solid angle per distance squared | |
| 2. measure/measurement of luminous flux, luminous power, i.e. “amount” of visible light emitted by a source, as measured by luminous intensity per solid angle | | 2. measure/measurement of photon flux, airglow, as measured in photons per distance squared per unit of time per solid angle | |
| 3. measure/measurement of luminance, i.e., brightness of light, as measured by luminous intensity per distance squared | | 3. measure/measurement of other illumination-based phenomena utilizing units of illumination-based measurement along with units of distance, time, power | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1/5$ derivative of Informal P3/S1 of the above root = candela; $SSD_1/9$ of Informal P3/S1 = candlepower
$SSD_1/5$ derivative of Informal P3/S2 of the above root = lumen
$SSD_1/5$ derivative of Informal P3/S3 of the above root = lux; $SSD_1/9$ of Informal P3/S1 = foot-candle

$SSD_1$ Derivatives for Formal Stem 1, Pattern 3:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | stilb | Degree 4 | | Degree 7 | apostilb |
| Degree 2 | foot-lambert | Degree 5 | candela per sq. meter = nit | Degree 8 | skot |
| Degree 3 | lambert | Degree 6 | | Degree 9 | bril |

$SSD_1$/5 derivative for Formal P3/S2 = rayleigh

$SSD_1$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | measurement of power of lens or eye, as measured in the reciprocal of distance (e.g., diopters) |
| Degree 3 | luminous energy, perceived energy of light, as measured by luminous intensity times unit of time per solid angle (e.g., talbots) |
| Degree 5 | luminous energy density, as measured by luminous intensity times unit of time per solid angle per cubic distance |
| Degree 7 | luminous exposure, as measured by luminous intensity times unit of time per distance squared (e.g., lux second) |
| Degree 9 | luminous efficacy, as measured by luminous intensity per solid angle per unit power (e.g., lumen per watt) |

| **-MPR-** MEASUREMENT OF AMOUNT OF SUBSTANCE / DENSITY | | | |
|---|---|---|---|
| 1. measure/measurement of chemical amount, i.e., amount of substance | | 1. measure/measurement of substance concentration, as measured by amount of substance per cubic distance | |
| 2. measure/measurement of density, as measured by mass per volume | | 2. measure/measurement of energy per amount of substance | |
| 3. measure/measurement of catalytic activity, as measured by amount of substance per unit time | | 3. measure/measurement of rate of other density-related phenomena utilizing units of density measurement along with distance, energy, temperature | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$/5 derivative of Informal P3/S1 = mole; $SSD_1$/9 derivative of P3/S1 = International Unit
$SSD_1$/5 derivative of Informal P3/S2 = grams per mole
$SSD_1$/5 derivative of Informal P3/S3 = moles per second (katal); $SSD_1$/9 derivative of P3/S3 = enzyme unit
$SSD_1$/5 derivative of Formal P3/S1 = mole per cubic meter
$SSD_1$/5 derivative of Formal P3/S2 = joule per mole

$SSD_1$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | volume occupied by an amount of a substance at a given temperature and pressure, as measured in cubic distance per amount of substance (e.g., molar volume = cu. meter per mole) |
| Degree 3 | ratio of the heat added to (or subtracted from) an object to the resulting temperature change, as measured in energy per unit temperature times amount of substance (e.g., molar heat capacity, molar entropy = joule per kelvin mole) |
| Degree 5 | efficiency of electrical conductivity of a substance, as measured by electrical conductance times square distance per amount of substance (e.g., molar conductivity = siemens times sq. meter per mole) |
| Degree 9 | chemical concentration, as measured by amount of substance per unit mass (e.g., molal = mole per kilogram) |

| **-PKL-** MEASUREMENT OF MISCELLANEOUS PHENOMENA | | | |
|---|---|---|---|
| 1. measure/measurement of statistical probability | | 1. measure/measurement of sound intensity, loudness | |
| 2. measure/measurement of content of information of an event based on the probability of the event | | 2. measure/measurement of acoustic absorption | |
| 3. measure/measurement of data transmission speed | | 3. measure/measurement of other miscellaneous phenomena | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement | Same as above stems with focus on act/process of measurement | Unit of measurement of said property; measure said property via a unit of measurement |

$SSD_1$/5 derivative of Informal P3/S1 of the above root = probit

$SSD_1$ Derivatives for Informal Stem 2, Pattern 3 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | dit | Degree 4 | | Degree 7 | shannon |
| Degree 2 | | Degree 5 | nat (nit, nepit) | Degree 8 | |
| Degree 3 | hartley (ban) | Degree 6 | | Degree 9 | bit |

$SSD_1$/5 derivative of Informal P3/S3 = baud

$SSD_1$ Derivatives for Formal Stem 1, Pattern 3:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | sone | Degree 4 | decibel | Degree 7 | phon |

$SSD_1$/5 derivative of Formal P3/S2 = sabin

$SSD_1$ Derivatives for Formal Stem 3, Pattern 1 of the above root:

| | |
|---|---|
| Degree 1 | osmotic pressure (e.g., osmol) |
| Degree 3 | quantized magnetic moment of a particle (e.g., nuclear magnetons, Bohr magnetons) |
| Degree 5 | fineness (purity) of precious metal based on ratio of the primary metal to any additives or impurities (e.g., karat) |
| Degree 9 | permeability of a porous material, as measured in distance squared (e.g., darcys) |

| **-TX-** MEDIUM OF EXCHANGE | | | |
|---|---|---|---|
| 1. medium of exchange (i.e., intermediary method of trade in avoidance of the inconveniences of a pure barter system); utilize a medium of exchange | | 1. act of financial accounting; to financially account | |
| 2. unit of money or currency | | 2. item of financial capital, financial instrument | |
| 3. quasi-contractual document equivalent to currency | | 3. commodity | |
| COMPLEMENTARY STEMS | | COMPLEMENTARY STEMS | |
| Medium of exchange as a concept in itself | Act of exchange utilizing a medium of exchange; engage in act of exchange utilizing a medium of exchange / trade using a medium of exchange | Act of financial accounting with focus on the means/process thereof | Act of financial accounting with focus on accurate economic evaluation of (one's) material/financial assets |
| coin | banknote | Bond, promissory note or equivalent | stock certificate or equivalent |
| cheque | scrip | Soft commodity (e.g., agricultural product) | Hard commodity (e.g., gold, oil) |

SSD derivatives for Informal Stem 2 of the above root are shown below for the 18 currencies associated with the most powerful world economies. Other units of currency can be referenced using Informal Stem 2 of the above root along with a the name of the nation or region in the CORRELATIVE or ORIGINATIVE case (as indicated by the carrier-root)

$SSD_1$ Derivatives for Informal Stem 2 (in Patterns 1, 2 or 3) of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | Chinese yuan | Degree 4 | Canadian dollar | Degree 7 | Japanese yen |
| Degree 2 | Russian ruble | Degree 5 | euro | Degree 8 | Brazilian real |
| Degree 3 | British pound | Degree 6 | Indian rupee | Degree 9 | U.S. dollar |

$SSD_2$ Derivatives for Informal Stem 2 of the above root:

| | | | | | |
|---|---|---|---|---|---|
| Degree 1 | Mexican peso | Degree 4 | Argentine peso | Degree 7 | Indonesian rupiah |
| Degree 2 | Swiss pound | Degree 5 | Australian dollar | Degree 8 | Saudi riyal |
| Degree 3 | Turkish lira | Degree 6 | Swedish krona | Degree 9 | South Korean won |

**The Expression Of Measurement In Ithkuil's Mathematical Sub-Language**

The affixes associated with units of measurement in Ithkuil's mathematical sub-language are shown in the charts below. Affixes associated with units of measurement consist of a consonantal form which will always include either a **-b-**, **-d-**, **-g-**, or an aspirated stop (**-p$^{h}$-**, **-t$^{h}$-**, **-k$^{h}$-**, **-q$^{h}$-**). Examples are **-lb-**, **-zd-**, **-gg-** preceded by a neutral vocalic increment **-ë-**.

Number affixes are NOT prefixed to the consonantal affix directly, but rather are prefixed to the vocalic increment **-ë-** preceding the consonantal increment of the unit of measurement affix.

**NOTE:** Readers are reminded that the basic meaning of Ithkuil unit-of-measurement stems are conceptual only and do not correspond to arbitrary units from Western systems of measurement. It is only the use of the **SSD** affixes associated with such stems by which Ithkuil units of measurement correspond to specific (and arbitrary) units such as SI or CGS units. In fact, the mathematical sub-language supports specification of units of measurement corresponding to all 27 **SSD**-affixed forms of an Ithkuil unit-of-measurement. This is accomplished by shifting the default vocalic **-ë-** increment of the affix as shown below:

- To specify the **SSD$_1$/1**-affixed form of an Ithkuil unit-of-measurement change the vocalic portion of the affix from **ë-** to **i-**.
- For **SSD$_1$/2**, change the vocalic increment from **ë-** to **ö-**.
- For **SSD$_1$/3**, change the vocalic increment from **ë-** to **e-**.
- For **SSD$_1$/4**, change the vocalic increment from **ë-** to **î-** or **û-**.
- For **SSD$_1$/5**, change the vocalic increment from **ë-** to **a-**.
- For **SSD$_1$/6**, change the vocalic increment from **ë-** to **â-**.
- For **SSD$_1$/7**, change the vocalic increment from **ë-** to **o-**.
- For **SSD$_1$/8**, change the vocalic increment from **ë-** to **ü-**.
- For **SSD$_1$/9**, change the vocalic increment from **ë-** to **u-**.

- To specify the **SSD$_2$/1**-affixed form of an Ithkuil unit-of-measurement change the vocalic portion of the affix from **ë-** to **iu-** or **ea-**.
- For **SSD$_2$/2**, change the vocalic increment from **ë-** to **öi-** or **öu-**.
- For **SSD$_2$/3**, change the vocalic increment from **ë-** to **ei-** or **eu-**.
- For **SSD$_2$/4**, change the vocalic increment from **ë-** to **io-**.
- For **SSD$_2$/5**, change the vocalic increment from **ë-** to **ai-** or **au-**.
- For **SSD$_2$/6**, change the vocalic increment from **ë-** to **ae-**.
- For **SSD$_2$/7**, change the vocalic increment from **ë-** to **oi-** or **ou-**.
- For **SSD$_2$/8**, change the vocalic increment from **ë-** to **üa-**.
- For **SSD$_2$/9**, change the vocalic increment from **ë-** to **ui-** or **oa-**.

For Suffixes composed of the SSD$_1$/5 suffix followed by an additional SSD$_1$ suffix change the vocalic portion of the 2$^{nd}$ SSD$_1$ affix as follows:

- For **SSD$_1$/1,** change the vocalic increment from **ë-** to **ia-**
- For **SSD$_3$/2**, change the vocalic increment from **ë-** to **öa-**.
- For **SSD$_3$/3**, change the vocalic increment from **ë-** to **ie-**.
- For **SSD$_3$/4**, change the vocalic increment from **ë-** to **ue-**.
- For **SSD$_3$/5**, change the vocalic increment from **ë-** to **eo-**.
- For **SSD$_3$/6**, change the vocalic increment from **ë-** to **ao-**.
- For **SSD$_3$/7**, change the vocalic increment from **ë-** to **oe-**.
- For **SSD$_3$/8**, change the vocalic increment from **ë-** to **uo-**.
- For **SSD$_3$/9**, change the vocalic increment from **ë-** to **ua-** .

To convey equivalence to English "per" as in "per meter", prefix **ř-** immediately before the consonantal portion of the affix unless the consonantal portion of the affix begins with **l-**, **m-** or **n-** or **ň-**. In these latter cases, no prefix **ř-** is used; instead, if the consonantal portion of the affix begins with **l-**, change the **l-** to **r-**; if it begins with **m-** or **n-**, change it to **ň-**; if it begins with **ň-**, change it to **m-**.

**NOTE**: The vocalic affixes above indicating specific SSD delineations must be distinguished from the standard vocalic affixes used with the mathematical sub-language (e.g., **-a-** for multiplication, **-i-** for addition, **-e-** for exponentiation, etc.). When a vocalic affix immediately precedes a consonantal affix denoting a unit of measurement, the interpretation of the vocalic affix

should be as an SSD delineation (or neutral -**ë**-). Use of standard vocalic affixes in conjunction with a unit of measurement must be separated from the unit of measurement affix by an epenthetic -**h**-.

**SPATIO-TEMPORAL OR DIMENSIONAL PROPERTIES**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| distance; length | b | Informal P3/S1 of -**KP$^{h}$**- |
| time | d | Formal P3/S1 of -**KP$^{h}$**- |
| planar angle | fw | Informal P3/S2 of -**KP$^{h}$**- |
| volumetric (solid) angle | fy | Informal P3/S3 of -**KP$^{h}$**- |
| frequency | dv | Formal P3/S2 of -**KP$^{h}$**- |
| non-cyclic/aperiodic activity | žd | Formal P3/S3 of -**KP$^{h}$**- |

***see each stem's SSD derivatives for list of specific units of measurement corresponding to SI, CGS, and other arbitrary units, e.g., meters, feet, parsecs, etc.

**AREA, VOLUME, VELOCITY AND ACCELERATION**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| planar area | mb | Informal P3/S1 of -**RK$^{h}$**- |
| spatial volume | lb | Informal P3/S2 of -**RK$^{h}$**- |
| speed, velocity | ld | Formal P3/S1 of -**RK$^{h}$**- |
| acceleration | nd | Formal P3/S2 of -**RK$^{h}$**- |

***see each stem's SSD derivatives for list of specific units of measurement corresponding to SI, CGS, and other arbitrary units, e.g., sq. feet, sq. meters, etc.

The following measurements are indicated by a combination of affixes in the mathematical sublanguage.

| | | |
|---|---|---|
| angular velocity; angular frequency | -fwëřd | $SSD_1$/1 of Formal P1/S3 of -**RK$^{h}$**- |
| jerk, jolt, surge, lurch | -bëřdeţ | $SSD_1$/2 of Formal P1/S3 of -**RK$^{h}$**- |
| snap, jounce | -bëřdep | $SSD_1$/3 of Formal P1/S3 of -**RK$^{h}$**- |
| kinematic viscosity, diffusivity coefficient | -bekëřd | $SSD_1$/5 of Formal P1/S3 of -**RK$^{h}$**- |
| volumetric flow | -beţëřd | $SSD_1$/7 of Formal P1/S3 of -**RK$^{h}$**- |
| spread rate by volume | -beţëřb | $SSD_1$/9 of Formal P1/S3 of -**RK$^{h}$**- |

**ENERGY, FORCE, PRESSURE, POWER**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| mass | g | Informal P3/S1 of **-KPL-** |
| atomic mass | lt$^{h}$ | $SSD_1$/2 of Informal P3/S1 of -**KPL**- |
| energy, work | bv | Informal P3/S2 of **-KPL-** |
| force | zd | Informal P3/S3 of **-KPL-** |
| pressure, stress | zb | Formal P3/S1 of **-KPL-** |
| power | gr | Formal P3/S2 of **-KPL-** |

***see each stem's SSD derivatives for list of specific units of measurement corresponding to SI, CGS, and other arbitrary units, e.g., grams, pounds, ounce, etc.

The following measurements are indicated by a combination of affixes in the mathematical sublanguage:

| | | |
|---|---|---|
| Line density | -gëňb | $SSD_1$/1 of Formal P1/S3 of **-KPL-** |
| volumetric density | -gërb | $SSD_1$/2 of Formal P1/S3 of **-KPL-** |
| specific volume | -lbëřg | $SSD_1$/3 of Formal P1/S3 of **-KPL-** |
| spread rate | -gëňb | $SSD_1$/4 of Formal P1/S3 of **-KPL-** |
| area density | -gëňb | $SSD_1$/5 of Formal P1/S3 of **-KPL-** |
| momentum, impulse | -zdëd | $SSD_1$/6 of Formal P1/S3 of **-KPL-** |
| angular momentum | -bvëd | $SSD_1$/7 of Formal P1/S3 of **-KPL-** |
| thrust | -zdëb | $SSD_1$/8 of Formal P1/S3 of **-KPL-** |

| | | |
|---|---|---|
| torque (or moment) | -zdëb | $SSD_1$/9 of Formal P1/S3 of **-KPL-** |
| yank | -zdëřd | $SSD_2$/1 of Formal P1/S3 of **-KPL-** |
| rate of absorbed dose of ionizing radiation | -bvëřgëřd | $SSD_2$/2 of Formal P1/S3 of **-KPL-** |
| specific energy, energy per unit mass | -bvëřg | $SSD_2$/3 of Formal P1/S3 of **-KPL-** |
| radiant exposure of a surface, energy distribution over a surface, insolation, solar radiation | -bvëřbek | $SSD_2$/4 of Formal P1/S3 of **-KPL-** |
| energy density | -bvëřbeţ | $SSD_2$/5 of Formal P1/S3 of **-KPL-** |
| surface tension | -zdëřb | $SSD_2$/6 of Formal P1/S3 of **-KPL-** |
| stiffness | -zdëřb | $SSD_2$/7 of Formal P1/S3 of **-KPL-** |
| dynamic viscosity | -zbëd | $SSD_2$/8 of Formal P1/S3 of **-KPL-** |
| acoustic impedance | -zbëdëřb | $SSD_2$/9 of Formal P1/S3 of **-KPL-** |
| spectral flux by frequency | -grëřdv | $SSD_1$/5 + $SSD_1$/1 of Formal P1/S3 of **-KPL-** |
| spectral flux by wavelength | -grëřb | $SSD_1$/5 + $SSD_1$/2 of Formal P1/S3 of **-KPL-** |
| heat flux density, irradiance, radiant exitance, radiosity | -grëňb | $SSD_1$/5 + $SSD_1$/3 of Formal P1/S3 of **-KPL-** |
| spectral exitance/radiosity/irradiance by frequency; strength of radio wave emission | -grëňbëřdv | $SSD_1$/5 + $SSD_1$/4 of Formal P1/S3 of **-KPL-** |
| spectral exitance/radiosity/irradiance by wavelength | -grëňbëřb | $SSD_1$/5 + $SSD_1$/5 of Formal P1/S3 of **-KPL-** |
| radiant intensity | -grëřfy(ë) | $SSD_1$/5 + $SSD_1$/6 of Formal P1/S3 of **-KPL-** |
| spectral intensity | -grëřfyëřdv | $SSD_1$/5 + $SSD_1$/7 of Formal P1/S3 of **-KPL-** |
| radiance | -grëřfyëňb | $SSD_1$/5 + $SSD_1$/8 of Formal P1/S3 of **-KPL-** |
| spectral radiance by frequency | -grëřfyëňbëřdv | $SSD_1$/5 + $SSD_1$/9 of Formal P1/S3 of **-KPL-** |

**TEMPERATURE AND THERMODYNAMIC PHENOMENA**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| thermodynamic temperature | bb | Informal P3/S1 of **-JG-** |

***see each stem's SSD derivatives for list of specific units of measurement corresponding to SI, CGS, and other arbitrary units, e.g., kelvins, degrees °F, etc.

The following measurements are indicated by a combination of affixes in the mathematical sublanguage:

| | | |
|---|---|---|
| heat capacity, entropy | -bvëřbb(ë) | Informal P3/S2 of **-JG-** |
| Specific heat capacity, specific entropy | -bvëřgëbb(ë) | Formal P3/S1 of **-JG-** |
| thermal resistance | -bbëřgr(ë) | Formal P3/S2 of **-JG-** |
| thermal conductivity | -grëřbëbb(ë) | Formal P3/S3 of **-JG-** |

**ELECTRO-MAGNETIC PHENOMENA**

| PHYSICAL PROPERTY BEING MEASURED | MATHEMATICAL SUBLANGUAGE AFFIX | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| electric current | dd | Informal P3/S1 of -**MP$^{h}$**- |
| electrical charge of specific intensity over period of time | gv | Informal P3/S2 of -**MP$^{h}$**- |
| electrical potential difference and electromotive force | gz | Informal P3/S3 of -**MP$^{h}$**- |
| magnetic flux | gl | Formal P3/S1 of -**MP$^{h}$**- |
| electrical capacitance | zg | Formal P3/S2 of -**MP$^{h}$**- |
| electrical inductance | žg | Formal P3/S3 of - **MP$^{h}$**- |
| electrical resistance of circuits | bg | Informal P3/S1 of -**ŇČ$^{h}$**- |
| electrical conductance | bn | Informal P3/S2 of -**ŇČ$^{h}$**- |
| magnetomotive force | gdh | Informal P3/S3 of -**ŇČ$^{h}$**- |
| magnetic flux density or B-magnetic field strength | gn | Formal P3/S1 of -**ŇČ$^{h}$**- |
| relative difference, i.e., ratio between two values of a physical quantity, e.g. power, intensity, current, voltage, loss or gain of an electronic signal, etc. | gm | Formal P3/S2 of -**ŇČ$^{h}$**- |

***see each stem's SSD derivatives for list of specific units of measurement corresponding to SI, CGS, and other arbitrary units, e.g., coulomb, tesla, gauss, etc.

The following measurements are indicated by a combination of affixes in the mathematical sublanguage:

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN |
|---|---|---|
| reciprocal inductance, reluctance | -ëržg | $SSD_1$/1 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electric field | -gzërb / -zdërgv | $SSD_1$/2 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electric displacement field, polarization vector | -gvëňb | $SSD_1$/3 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electric charge density | -gvërbeţ | $SSD_1$/4 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electric current density | -ddëňb | $SSD_1$/5 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electrical resistivity | -bgëb | $SSD_1$/6 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electrical conductivity | -bnërb | $SSD_1$/7 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electromagnetic emittivity | -zgërb | $SSD_1$/8 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electromagnetic permeability | -žgërb | $SSD_1$/9 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| electric elastance | -ërzg | $SSD_2$/1 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| H-magnetic field strength | -ddërb | $SSD_2$/5 of Formal P1/S3 of -**ŇČ$^{h}$**- |
| exposure to ionizing radiation | -gvërg | $SSD_2$/9 of Formal P1/S3 of -**ŇČ$^{h}$**- |

**ILLUMINATION**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| luminous intensity | bz | Informal P3/S1 of -**CTW**- |
| luminous flux, luminous power, i.e. "amount" of visible light emitted by a source | bl | Informal P3/S2 of -**CTW**- |
| illuminance, luminous exitance or emittance | br | Informal P3/S3 of -**CTW**- |
| luminance, i.e., brightness of light | bž | Formal P3/S1 of -**CTW**- |
| photon flux, airglow | bd | Formal P3/S2 of -**CTW**- |

***see each stem's SSD derivatives for specific units of measurement corresponding to SI, CGS, and other arbitrary units, e.g., candelas, candlepowers, etc.

The following measurements are indicated by a combination of affixes in the mathematical sublanguage:

| | | |
|---|---|---|
| measurement of power of lens or eye | -ërb | $SSD_2$/1 of Formal P1/S3 of -**CTW**- |
| luminous energy, perceived energy of light | -blëd | $SSD_2$/3 of Formal P1/S3 of -**CTW**- |
| luminous energy density | -blëdërbeţ | $SSD_2$/5 of Formal P1/S3 of -**CTW**- |
| luminous exposure | -brëd | $SSD_2$/7 of Formal P1/S3 of -**CTW**- |
| luminous efficacy | -blërgrë | $SSD_2$/9 of Formal P1/S3 of -**CTW**- |

**AMOUNT OF SUBSTANCE / DENSITY**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| amount of substance | gg | Informal P1/S1 of -**MPR**- |
| density of a substance | dg | Informal P1/S2 of -**MPR**- |
| catalytic activity | gd | Informal P1/S3 of -**MPR**- |

***see each stem's SSD derivatives for list of specific units of measurement corresponding to SI, CGS, and other arbitrary units, e.g., moles, katals, etc.

The following measurements are indicated by a combination of affixes in the mathematical sublanguage:

| | | |
|---|---|---|
| substance concentration or mass concentration of a pure substance | -ggërbeţ | Formal P1/S1 of -**MPR**- |
| energy per amount of substance | -bvërgg(ë) | Formal P1/S2 of -**MPR**- |
| volume occupied by an amount of a substance at a given temperature and pressure | -beţërgg(ë) | $SSD_1$/1 of Formal P1/S3 of -**MPR**- |
| ratio of the heat added to (or subtracted from) an object to the resulting temperature change | -bvërbbëgg(ë) | $SSD_1$/3 of Formal P1/S3 of -**MPR**- |
| efficiency of electrical conductivity of a substance | -bnërbekërgg(ë) | $SSD_1$/5 of Formal P1/S3 of -**MPR**- |
| chemical concentration | -ggërg | $SSD_1$/9 of Formal P1/S3 of -**MPR**- |

**MISCELLANEOUS PHENOMENA**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| content of information of an event based on the probability of the event | gž | Informal P3/S1 of **-PKL-** |
| statistical probability | dn | Informal P3/S2 of **-PKL-** |
| data transmission speed | žb | Informal P3/S3 of **-PKL-** |
| sound intensity, loudness | dr | Formal P3/S1 of **-PKL-** |
| acoustic absorption | gdh | Formal P3/S2 of **-PKL-** |
| osmotic pressure | bdh | $SSD_1$/1 of Formal P1/S3 of -**PKL**- |
| fineness (purity) of precious metal based on ratio of the primary metal to any additives or impurities | dl | $SSD_1$/3 of Formal P1/S3 of -**PKL**- |
| permeability of a porous material | gb | $SSD_1$/5 of Formal P1/S3 of -**PKL**- |
| quantized magnetic moment of a particle | db | $SSD_1$/9 of Formal P1/S3 of -**PKL**- |

**MEDIUM OF EXCHANGE**

| PHYSICAL PROPERTY BEING MEASURED | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN* |
|---|---|---|
| monetary currency | dm | Informal Stem 2 of -**TX**- |

**NOTE**: The following physical constants, although containing consonants (**b**, **d**, **g** and aspirated stops) that signify them as units of measurements, nevertheless function like numbers within mathematical expressions, i.e., vocalic affixes pertaining to mathematical operations can be directly prefixed to them without an intervening epenthetic -**h**- and they do not take the vocalic shifts signifying different degrees of an SSD suffix. (This explains why these constants were also listed earlier within mathematical expressions).

| PHYSICAL CONSTANT | AFFIX IN THE MATHEMATICAL SUBLANGUAGE | CORRESPONDING ITHKUIL ROOT/STEM/PATTERN |
|---|---|---|
| Avogadro's number | bm | $SSD_1$/1 of Formal P1/S3 of -**C**- |
| reduced Planck constant | gw | $SSD_1$/2 of Formal P1/S3 of -**C-** |
| Coulomb constant | by | $SSD_1$/3 of Formal P1/S3 of -**C**- |
| Universal gas constant | $nt^h$ | $SSD_1$/4 of Formal P1/S3 of -**C**- |
| speed of light in a vacuum | bw | $SSD_1$/5 of Formal P1/S3 of -**C**- |
| Rydberg constant | $mp^h$ | $SSD_1$/6 of Formal P1/S3 of -**C**- |
| Stefan-Boltzmann constant | dy | $SSD_1$/7 of Formal P1/S3 of -**C**- |
| universal gravitational constant | dw | $SSD_1$/8 of Formal P1/S3 of -**C**- |
| Acceleration due to gravity | gy | $SSD_1$/9 of Formal P1/S3 of -**C**- |

Examples of expressions using units of measurement:

**Istál ôk'àlâb iarwáiřl ët'ëgëbekut'ëřdek.**

*OR* **Istál ôk'àlâb epáil ëbv.**

*A unit of energy is measured by solving the equation* $\frac{\text{mass} \times \text{distance}^2}{\text{time}^2}$.

The above example illustrates that Ithkuil units of measurement, in and of themselves are conceptual only, irrespective of a particular arbitrary measurement system. Compare the above example with the example below, in which the various unit affixes are modified by vowels to correspond to the $SSD_1$/5 affix associated with their parent stems for measurement of mass, measurement of distance, and measurement of time:

**Istál ôk'àlâb iarwáiřl ët'agabekut'ařdek.**

*A unit of energy is measured by solving the equation* $\frac{\text{kilogram} \times \text{meter}^2}{\text{second}^2}$ .

Utilizing the $SSD_1/5$ affix assigned to the stem for measurement of energy, the above is equivalent to saying:

**Ëlëbv ¯âhëlábv.** *A unit of energy is one joule.*

Or one can restate the sentence utilizing different $SSD_1$ affixes to correspond with other systems of measurement:

**Ëlëbv ¯âhëlúbv.** *A unit of energy is one erg.*

**Ëlëbv ¯âhëlíbv.** *A unit of energy is one calorie.*

**Ëlëbv ¯âhëlébv.** *A unit of energy is one therm.*

Other examples:

**Ôk'ál ¯ëhâhëgabwék.** $E = mc^2$.

**Eilpalelb epèöl ¯ëlňánd.** *The vehicle is accelerating at twelve meters per second per second.*

**Eijjawelök îcmeöl ¯ëfwàxhôf.** *Turn (your body) 45 degrees eastward.*

**Eik'àlôpš ¯ëlkţablarunsàgrë.** *The lamp generates 171 lumens and 89 watts.*

**Olkal tô ëqëňšqiub ¯âlšmëňáb.** *I am 72½ inches tall, equal to 1.84 meters.* [60.6 inches and 1.A1 meters in base-12]